Essay in Dynamics

showing the wonderful laws of nature concerning bodily forces and their interactions,
and tracing them to their causes

G. W. Leibniz

[Brackets] enclose editorial explanations. Small ·dots· enclose material that has been added, but can be read as though it were part of the original text. Occasional •bullets, and also indenting of passages that are not quotations, are meant as aids to grasping the structure of a sentence or a thought. Every four-point ellipsis . . . . indicates the omission of a brief passage that seems to present more difficulty than it is worth.—Leibniz illustrates some of his points with five diagrams; but none of them is needed, and in each case the text is easier to follow when cleansed of the references to diagrams.—The division into numbered portions is mostly that of Woolhouse and Francks. The division into two Parts is original to this version.

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Part 1: Clearing away errors

1. Since I first mentioned the founding of a *new science of dynamics*, many distinguished people in various places have asked me to explain this doctrine more fully. So, not having had time to write a book ·about it·, I here offer something that may throw some light on the subject. That light may be returned with interest, if I can get the opinions of people who combine power of thought with elegance of expression. Their judgment would be very welcome and, I hope, useful in advancing the project.

2. I have proclaimed elsewhere that there is more to bodily things than extension—indeed that there is in them something prior to [= 'more basic than'] extension, namely the *force* of nature that God has given to everything. This force isn't a mere faculty ·or ability· of the kind the Aristotelians seem to be satisfied with; rather, is equipped with a striving [*conatus*] or effort [*nisus*] such that the force will have its full effect unless it is blocked by some contrary striving. [Striving is just *trying*, of course; but for Leibniz *conatus* and the related verb are technical terms in physics, and in this text 'strive' etc. will always be used for that technical notion.] We are often sensorily aware of this effort, and reason shows—I maintain—that it is everywhere in matter, even when the senses don't detect it. We shouldn't attribute this force [*vis*] to God's miraculous action (·i.e. to his pushing around of bodies that in themselves are inert·); so it is clear that he must have put it into the bodies themselves—indeed, that it constitutes the inmost nature of bodies. For what makes a substance a *substance* is that it *acts*. Mere extension doesn't make something a substance; indeed extension *presupposes* a substance, one that exerts effort and resistance; extension is merely the continuation or spreading-out of that substance.

3. ·You might object that· all bodily action arises from motion, and that motion itself comes only from other motion either in the moving body itself or in something else that collides with it. But that doesn't affect the matter. ·It couldn't be that *motion* is the fundamental category in physics, because· motion, when we analyse it, doesn't really exist; for a whole doesn't exist if it doesn't have coexistent parts (time also doesn't really exist, for the same reason). So all that is real in motion is a momentary state that must be produced by a force that strives for change. That is all there is to bodily nature: •extension (·the set of properties that are· the subject of geometry) and •this force. ·The next two sections are an aside, though an important one·.

4. This theory at last does justice both to the truth and to the teaching of the ancients. Just as our age has already rescued from scorn Democritus's atoms, Plato's ideas, and the Stoics' tranquility about the best possible arrangement of things, so we can restore the Aristotelian doctrine of forms or 'entelechies' to the ranks of *intelligible notions*, though it has rightly struck people as puzzling, and wasn't understood properly even by its own inventors. This philosophy ·of forms·, it seems to me, shouldn't be tossed aside after having been accepted for so many centuries; rather, it should be explained in such a way as to make it self-consistent (where possible), and should be extended and illustrated with new truths.

5. This approach to inquiry, ·in which the best of the old is combined with the best of the new·, strikes me as sensible for the teacher and useful for the student. It will •stop us from appearing more eager to destroy than to build, •save us from being continually bounced from one bold new theory to another, uncertain what to think, and •restrain the urge to form sects, an urge that is encouraged by the empty glory of novelty. This will enable mankind to establish secure doctrines, and to advance with firm steps towards greater heights, in philosophy and science as much as in mathematics. For the writings of distinguished men, both ancient and modern, if you set aside their scolding of one another, usually contain a great deal that is true and good, and that deserves to be dug up ·from obscure books· and displayed in the treasury of public knowledge. If only people preferred doing *this* to wasting time on criticisms that flatter their own vanity! I myself have had the good fortune to discover certain new ideas—so that my friends keep telling me to think about nothing else!—but I can still appreciate other opinions, even hostile ones, judging them on their own differing merits. Perhaps that is because by working at a lot of things you learn to despise none of them. Now I return to my main thread.

[Leibniz will write about a force he calls *primitiva*, usually translated as 'primitive'. The force is not 'primitive' in any now-current sense of that word; *primitiva* means 'early, first-formed'; as applied to a *force* the best translation seems to be 'basic'.]

6. Active force (which some not unreasonably call 'power ') is of two kinds. There is •*basic* active force and •*derivative* active force. •Basic active force is present in all bodily substance, just because it is bodily substance; it would be contrary to the nature of things for there to be a body that was wholly at rest, ·which a body *would* be if it had no inherent active force·. •Derivative active force is what becomes of basic force when bodies collide with one another. . . . Basic force—which is no other than the 'first entelechy' ·that Aristotle theorized about·—corresponds to the *soul* or *substantial form*; but just for that reason it relates only to general causes, which aren't enough to explain ·specific kinds of· phenomena. So I agree with those who say that we shouldn't appeal to 'forms' in explaining the causes of things that we experience. I need to point this out, so that when I try to •restore to 'forms' their lost right to be counted among the ultimate causes of things I don't seem to be also trying to •revive the verbal disputes of the second-rate Aristotelians. But some knowledge of forms is necessary for doing philosophy ·and science· properly. No-one can claim to have properly understood the nature of body unless he has •thought about such things, and has •understood what is incomplete and false in a certain crude notion of bodily substance. The one I mean is based entirely on sensory ideas, and was rashly introduced into the corpuscular philosophy—which in itself is most excellent and true—some years ago. This—·its inadequacy·—is shown by the fact that it can't rule out matter's being completely inactive or at rest, and can't explain the laws of nature that govern derivative force.

7. Passive force is similarly of two kinds, •basic and •derivative. When the Aristotelians write about 'primary matter', they make best sense when understood to be referring to the basic force of being acted on—the basic force of resistance. It explains ·the following three facts·:

> •Bodies, rather than interpenetrating ·so that two bodies occupy the same place at the same time·, *block* one another.
>
> •Bodies have a certain laziness, as it were—a reluctance to move.
>
> •No body will allow itself to be set in motion ·in a collision· without somewhat lessening the force of the body acting on it.

The derivative force of being acted on shows up after that in various ways, in ·what the Aristotelians have called· *secondary matter*.

8. From these basic, general points we learn that all bodies always •act by virtue of their •form, and are always •acted upon and resist because of their •matter. Having expounded this, I must push on into the doctrine of *derivative powers and resistances*, showing how bodies act on and resist each other to differing extents according to their different levels of effort. These things are covered by the laws of action—laws that are not only understood through reason but also confirmed by experience of the phenomena.

9. By 'derivative force'—the force by which bodies actually act and are acted on by each other—I mean here just the force that is involved in motion and which in turn tends to produce further motion. (·Here and throughout·, I mean *local* motion, of course; ·that is, motion from place to place, not the mere *alterations* that some philosophers have called '(nonlocal) motion'·.) For I realize that all other phenomena involving matter can be explained in terms of ·local· motion. Motion is continuous •change of place, and so requires •time. But while

> it is •through time that a movable thing •moves,
> it is •at individual moments that it •has a velocity,

the velocity being greater as the thing covers more space in less time. ·Two technical terms have to be introduced here·.

> •Conatus [= 'striving'] is velocity taken together with direction.
>
> •Impetus is the product of the bulk of a body and its velocity.

10. Impetus is the quantity that the Cartesians usually call the 'quantity of motion'—that is, the quantity of motion at a moment. Though, to speak more accurately, the quantity of motion actually exists over time, and is the sum of the products of •the different impetuses existing in the moving thing at different times and •the corresponding time intervals. In arguing with the Cartesians I have followed this inaccurate terminology of theirs, ·but I need to clean it up here·. A technically convenient distinction lets us distinguish •an increase that is occurring right now from •one that has occurred or is going to occur; and we can speak of the former as an increment or 'element' of the ·whole time-taking· increase. It also lets us distinguish the present falling of a body from the fall which has been going on and is increasing. In the same spirit we can distinguish •the present or instantaneous 'element' of motion from •the motion itself taken as extended over time. If we call the former 'point motion', then ·impetus, which is· what Cartesians and others call 'quantity of motion', can be called 'quantity of point motion'. We can be slack in our use of words once we have given them a precise meaning, but until then we must use them carefully so as not to be led astray by ambiguities. [Of Latin's two words for 'motion', Leibniz used *motus* for (time-taking) motion, and stipulated that *motio* was to stand for the instantaneous item that is here called 'point motion'.]

11. Just as the numerical value of a •motion stretching through time is derived from an infinite number of •impetuses, so in turn an •impetus, even though it is momentary, is derived from an infinite series of 'elements' imparted to the moving body. So an impetus, like a time-taking motion·, can come into existence only through an infinite repetition of a certain element that it contains. ·Here is evidence for this·. Imagine

> a tube anchored at one end and whirling around at a uniform speed in a circle on the horizontal plane of this page,

and

> a ball inside the tube, held in position down near the anchored end.

Consider •the instant when the ball is released from the restraint that has been holding it in position. At that instant, the ball has two impetuses:

a *considerable* rotational one, giving it a tendency to move around with the tube, keeping the same distance from the anchor,

and

an *infinitely smaller* centrifugal one giving it a tendency to move to the end of the tube away from the anchor.

Now consider •an instant half a second later: the ball has been carried *with* the tube some distance around its circle, and has also moved *along* the tube towards its outer end, the end away from the anchor. At that second instant, the ball has a considerable centrifugal impetus (how big it is depends on how fast the tube is making it rotate). It is obvious from this that effort [always translating *nisus*] is of two kinds:

•infinitely small efforts, which are 'elements' of the larger kind; I also call an instance of this kind of effort an 'urge' [Latin *solicitatio*]

and

• the impetus itself, which is built up by the continuation or repetition of the *elementary* efforts.

I don't infer from this that these mathematical entities are really to be met with in nature—only that they are useful as mental abstractions for making accurate calculations.

12. Force, therefore, is also of two kinds. •One is elementary, and I call it *dead force*, since there is no motion in it yet but, only an urge to motion like that of the ball in the tube. . . . •The other is just ordinary force, which is accompanied by actual motion. I call it *live force*. Examples of dead force:

•centrifugal force
•the force of gravity or centripetal force
•the force by which a stretched elastic body tries to spring back into shape.

Examples of living force, which arises from an infinity of continued impulses of dead force, are provided by

•the impact arising from a heavy body that ·hits the ground after it· has been falling for some time,
•a bent bow that is part of the way through springing back into shape.

and the like. This is what Galileo meant when he said rather enigmatically that the force of impact is infinite as compared with the mere effort of gravitational force. However, even though impetus is always accompanied by living force, I shall show below that the two are different.

13. Living force in a bodily aggregate—·that is, in a material thing with parts·—can also be understood as being of two kinds, *total* and *partial*. And partial living force also divides into:

•inward-acting or parts-only force, through which the parts of the aggregate can act on each other,

and

•directive or ordinary force, through which the aggregate as a whole can act on other things.

I call the latter 'directive' because this kind of partial force contains the whole force of the overall direction of the aggregate. Pretend that the aggregate has suddenly fused together, ·as though frozen·, so that its parts stop moving relative to each other: then directive force is the only force left. So *total absolute force* consists of inward-acting and directive force taken together. But this will be clearer from the rules presented below.

14. As far as we can tell, the ancients had disciplined knowledge only of dead force. This is what is commonly called 'mechanics'. It deals with levers, pulleys, inclined planes (including wedges and screws), the equilibrium of liquids, and similar matters. It considers only the first striving of individual bodies in themselves, before they acquire impetus through action. There is a way to bring the laws of dead force over to living force, but we have to be very cautious in doing this if we are not to be misled. Some people *were* misled in this way: knowing that *dead force* is proportional to the product of bulk and velocity, they concluded that force in general is proportional to that product, because they didn't make the needed distinction.

15. As I once pointed out, there is a special reason why that proportionality holds in that case—·that is, at the start of a body's fall·—as can be seen from this example: Suppose that bodies of different weights are falling; at the start of the fall of each, when the amount of space covered in the descent is infinitely small or 'elementary', the amount of descent is proportional to the speed or striving of descent. But when the fall has gone some distance, and a living force has developed, the acquired speed is no longer proportional to the distance fallen (which is, however, as I have shown before and will show again in more detail later, how the force should be measured). Rather, the speed is proportional to the 'elements' of those distances.

16. The treatment of living force began with Galileo, though he used a different name for it and, indeed, a different concept. He was the first to explain how motion arises from the acceleration of falling bodies. Descartes •correctly distinguished velocity from direction, and also •saw that what results from a collision between bodies is the state of affairs that involves the *least change* from the state of affairs before the collision. But he did not calculate that least change correctly: he changed the direction and then the velocity, separately, when the change should be determined by both at the same time. He didn't see how this could be done. He was concerned with modalities rather than realities—·i.e. with conceptual relationships, rather than real happenings in the world·—and he didn't see two such different things as direction and velocity could be compared and considered at the same time. (He committed other errors as well, but I shan't go into them.)

17. . . . .As far as I know, the first to arrive at the pure and simple truth on this question was Huygens, who has enlightened our age with his brilliant discoveries. He purged the subject of fallacies by means of the laws he published some time ago. Wren, Wallis, and Mariotte, all distinguished men in this field in their own different ways, have arrived at almost the same laws, but not at agreement as to the causes; so that even these men, outstanding as they are in these studies, don't always come to the same conclusions. It is clear, therefore, that the true basis of this science—which I have established—hasn't yet been revealed ·to physicists in general·. There isn't universal acceptance even of the proposition, which to me seems quite certain, that rebounding ·in collisions· arises purely from elastic force, that is, from resistance due to internal motion. And no-one before me has explained the notion of force itself. This has always been a problem for the Cartesians and others, who couldn't understand that the sum total of •motion or impetus—which they took to be the amount of force—could be different after a collision from what it was before, because they thought that would mean that the quantity of •force would be different as well.

18. In my youth I agreed with Democritus (and also with Gassendi and Descartes, who in this respect follow him) that the nature of body consists only in inert *mass*, and ·at the age of 25· I put out a little book called *A New Physical Hypothesis*, in which I offered a theory of motion—presented both in abstract theoretical terms and in concrete applications ·to the real world·. Some distinguished men seem to have liked this book rather more than its mediocrity deserves. In it I showed that given this conception of body ·as an inert mass·, a moving body will pass its striving on to whatever body it collides with. . . . For at the moment of collision it strives to continue its motion, and so strives to carry the other body along with it, and *this striving must have its full effect on the body collided with unless it is prevented by an opposing striving,* and even if it *is* opposed, this will still hold true, because the different conatuses ·of the two bodies· will then have to be combined. So there was no reason why the impacting body should not achieve the effect it was tending towards, or why the other body should not receive the full striving of the impacting body, so that the motion of the other body ·after the collision· would be the combination of its own original striving and the new striving received from outside. (*·In the asterisked bit of this argument· I was relying on the view I held at that time, that bodies are indifferent to motion and rest. ·More about this shortly·.)

19. [This section is being handled rather freely; the system of ·dots· can't conveniently be used.] From this I also showed that on the following assumptions—

- •There is nothing to body except its mathematical properties—size, shape, position—and their changes, together with a striving for change which it has at the moment of impact *and only then*;
- •Nothing is to be explained through metaphysical notions such as active power in the *form* and sluggishness or resistance to motion in *matter*; and therefore
- •The outcome of a collision has to be determined purely by the geometrical composition of strivings, as just explained;

—given those assumptions, it follows that a pea could knock away a cannon-ball! The whole striving of the impacting body (the pea) would be passed on to the body it collided with (the cannon-ball), which would be carried away by the collision with the pea, which wouldn't even be slowed down. That is because matter, on those assumptions about it, has no resistance to motion, but is wholly indifferent to it. This implies that •it would be no harder to move a large body than to move a small one, and therefore •that there would be action without reaction, and •there would be no measure of power, because anything could overcome anything! Because of these upshots and many others that also go against the order of things and conflict with the principles of true metaphysics, I concluded (rightly!) that in creating the system of things the all-wise creator had been careful to avoid the consequences which would have followed from the skimpy laws of motion that you get if you look only to geometry, ·as Descartes did, and as I did at the age of 25·.

20. Later on, when I looked more deeply into this, I came to see what a systematic explanation of things would consist in, and I realized that my earlier theory about the notion of body was incomplete. By means of the above argument and some others, I was able to establish that we have to posit in bodies something more than mere size and impenetrability—something that brings in considerations of *force*. When the metaphysical laws of this 'something' are added to the laws of extension (·the ones you get just from geometry·), there arise what I call *systematic rules of motion*—

- •All change is gradual;
- •Every action also has a reaction;
- •No new force is produced without reducing an earlier one, so that a body that pushes away another body will be slowed down by it;
- •There is neither more nor less power in an effect than in its cause.

As this law isn't derivable from the concept of mass, it must come from something else in bodies, namely from **force**, of which the same quantity is always maintained though it may be carried by different bodies. I therefore concluded that in addition to what falls under pure mathematics and the imagination—·i.e. concepts that are exemplified in sense-experience·—we must accept something metaphysical that is perceptible only to the mind ·and not through the senses·; and that in addition to •material mass we must add some higher kind of principle that might be called •formal. [Leibniz is here invoking the traditional distinction between *form* and *matter*.] For not all truths about bodily things can be derived from logical and geometrical axioms alone, that is, from those pertaining to

large and small,
whole and part,
shape, and
position.

To explain the order of things properly we have to bring in other notions involving

cause and effect, and
activity and passivity.

It doesn't matter whether we call this principle 'form', or 'entelechy', or 'force', provided we remember that it can be intelligibly explained only through the concept of force. [In this section 'principle', translating Leibniz's *principium*, does not stand for anything propositional. Its various shades of meaning involve the likes of a *source*, a *start*, an *initial launching*, an *origin*, a *basic force*. In many contexts 'force' is a good translation, but obviously not here.]

21. Some distinguished contemporaries have grasped this ·important· fact that the usual concept of matter is not adequate, and have used it as their lead into •a denial that things have any force for action and •the introduction of God as a convenient cause ·who does all the things that material things seem to do but (according to these men) don't really do·. . . . I don't agree with this at all. They have clearly shown, I agree, that as a matter of strict metaphysics one created substance can't affect another by sending something across to it. [This refers to the view, held by some philosophers, that one thing can affect another by sending across to it an 'accident'—an instance of one of its properties. According to these philosophers, in addition to the •universal property *heat* and the •particular thing *this poker* there is a •particular property, an instance, an accident, namely *the heat of this poker*; and they held that when the poker is plunged into cold water it sends an accident—some of its particular heat—across to the water. Leibniz is agreeing here with his present opponents that *this* account of how things interact won't do.] And I willingly admit that whatever happens has underlying it God's continual creation, ·i.e. his continual activity of keeping his creations in existence·. But ·against these people who invoke God's interference to explain the seeming interactions between bodies· I hold that none of the natural facts about things should be immediately explained in terms of what God does or wants, and that God has endowed each thing with something through which all its predicates can be explained. ·So the thing's behaviour is to be explained in terms of *its own* properties and powers; God gave them to it, of course, but he doesn't have to come into the immediate explanation·. . . .

22. Meanwhile, although I hold that all bodies contain an active or (so to speak) *vital* force which stands above all material concepts, I don't agree with Henry More and other men of outstanding piety and intelligence who appeal to bizarre spiritual forces to explain the phenomena—as if

- •not everything in nature could be explained mechanically, and
- •those who try to give mechanistic explanations are denying the existence of things that aren't bodies, exposing themselves to a suspicion of impiety, or

- •we ought to attribute *minds* to the rotating the spheres, as Aristotle did, or to say that the rising and falling of the elements is due to their 'forms'—a nice neat theory that tells us nothing.

[Leibniz is soon going to speak of 'efficient' and 'final' causes. The efficient cause of something is what *makes* it happen; it is simply what we today would call a 'cause' (with no adjective). A thing's final cause is its purpose or end, what it is *for*.]

23. I do not, as I say, agree with these theories. I don't like this approach any more than I do the theology of people who were so sure that Jupiter causes thunder and snow that they levelled charges of atheism at anyone who tried to find more specific causes of such things! I think it is best to take a middle path, satisfying both •science and •religion:

> I accept that •all bodily phenomena can be traced back to mechanical efficient causes, but we are to understand that •those mechanical laws as a whole derive from higher reasons;

so we *can* appeal to higher efficient causes—·namely, to the actions of God·—but only in establishing those remote and general explanations, ·not in explaining particular phenomena·. Once those general laws have been established, entelechies or souls have no place in discussions of the immediate and specific efficient causes of natural things, any more than do useless 'faculties' and inexplicable 'sympathies'. The first and most universal efficient cause should not be considered in tackling specific problems, except when we contemplate the purposes that God in his wisdom had in ordering things in that way—which we may do in order not to miss any opportunity for praising him and singing lovely hymns.

·AN ASIDE·: In fact, *final* causes can sometimes also be introduced with great profit into some particular problems in physics—not just so that we can better admire God's most beautiful works (·as mentioned above·) but also sometimes in order to discover ·more specific· things that the *efficient*-cause approach would have more difficulty establishing, or could establish only with extra assumptions. I showed this by a quite remarkable example of a principle in optics, which the famous Molyneux applauded in his *New Optics*. [This refers to Leibniz's account of a law relating the angle at which light impinges on a piece of glass (say) to the angle at which it leaves it. This law, he says, has the effect that the light travels to some given point on the far side of the glass by 'the easiest way', and he thinks that this involves the final-cause concept. Apparently this is not meant to invoke •God's purpose in arranging things thus, but rather light's behaving as though *it* had the purpose of reaching the given point by the easiest way.] Perhaps scientists haven't yet taken in how useful this kind of appeal to final causes can be. ·It is different from the kind mentioned in **23** and **24**, which is why this is an aside, now ending·.

24. In general we should hold that everything can be explained in two ways: in terms of the •kingdom of power, or *efficient* causes, and in terms of the •kingdom of wisdom, or *final* causes. God governs bodies in the way a designer governs machines, in accordance with the laws of geometry; but he does so for the benefit of souls. And souls, which are capable of wisdom, he governs for his greater glory as citizens or fellow members of society, in the manner of a prince or a father, in accordance with laws of goodness or of morality. These two kingdoms thoroughly interpenetrate each other without any mixing or disturbing of their laws, so that there arises the •*greatest* in the kingdom of •power along with the •*best* in the kingdom of •wisdom. But my task here is to establish the general rules for effective forces, which we can then use to explain particular efficient causes.

25. Next, I worked out how to measure forces correctly, and in two very different ways. One is *a priori*, from the simplest consideration of space, time and action; I shall explain this elsewhere. The other is *a posteriori*; it measures a force by the effects it produces in expending itself. By 'effect' here I don't mean *any* old effect, but one that needs the expenditure of force, one in which the force is all used up; such an effect could be called •*violent*. The effect produced by a heavy body in moving along a perfectly horizontal plane is not violent, because in this case the force is retained, ·rather than used up·, however long the effect goes on. Such effects might be called •*harmless*. They can be measured by my method, but I shall ignore them here.

26. The particular kind of violent effect that I have chosen is the one that is most homogeneous, that is, the most capable of being divided into ·qualitatively· similar and ·quantitatively· equal parts—such as the upward motion of a heavy body. ·This is homogeneous· because the ascent of a heavy body to two (or three) feet is exactly two (or three) times the ascent of the same body to one foot; and the ascent of a heavy body measuring 200 cm^3 to a height of one foot is exactly twice that of a heavy body measuring 100 cm^3 to that same height. So the ascent of a body measuring 200 cm^3 to three feet is exactly six times the ascent of a body measuring 100 cm^3 to one foot. For ease of exposition, I am assuming that heavy bodies weigh the same whatever height they are at—which in fact is not true, but the error is imperceptible. (Elastic bodies do not so easily lend themselves to considerations of homogeneity.)

27. Thus, to compare bodies with different sizes and different speeds, I easily saw that if body B is twice the size of body A and they are moving at the same speed, A would have one unit of force and B would have two units: B must have exactly twice what there is in A, because the only difference between them is that B is twice the size of A. But I saw that if bodies A and C are the same size but C's speed is twice that of A, it doesn't follow that C has exactly twice ·the amount of force· that A has—since what is doubled in C is not A's size but only its speed. This led me to see that a mistake had been made at this point by people—·such as the Cartesians·—who think that force is doubled merely by this kind of doubling of either size or speed. (·In this Essay I haven't yet given any reason for this judgment. I shall make the case for it in **29** and **30**·.)

28. I pointed this out some time ago. . . .and warned that the right technique for measuring forces involves finally getting down to something homogeneous; that is, to something allowing of accurate and complete duplications—both in *things* (·this body is exactly the size of that one·) and in their states (·this body is moving at exactly the same speed as that one·).There is no better or more noteworthy example of this technique than the one given by this proof ·that I now present·.

29. In order to obtain a measure of force, I asked this question:

> Given that bodies A and C are equal in size but different in speed, could they produce any effects that were equal in power to their causes and were homogeneous with each other?

If the answer was Yes, the forces of A and C—·though not directly comparable·—could still be accurately compared by means of their effects. (I assumed that an effect must be equal ·in power· to its cause if the whole power of the cause is used up in producing it, no matter how quickly or slowly this takes place.) ·And it turned out that the answer to the question *was* Yes, as you can see from what follows·.

30. Let A and C be bodies of ·equal size and· one pound in weight, with C moving at twice the speed of A. Let the forces involved in those motions be directed to a change of *height* - for example by letting A and C each be a weight at the end of a pendulum (the pendulums being of equal length). Now the demonstrations of Galileo and others have established that if body A rises to a point **one** foot above its lowest point, then body C, going at **twice** the speed, will rise to **four** feet above its lowest point. Raising one pound four feet is *exactly* the same as raising one pound one foot *four times*, which means that in this experiment C has done exactly four times as much as A, which has only raised one pound one foot *once*. So we find that **doubling** the speed **quadruples** the capacity for action, because when the double-speed C expends all its power it does four times as much. And in the same way we can conclude generally that the forces of equal bodies are proportional to the squares of their speeds and that, in general, *a body's force is proportional to the product of its size and the square of its speed.*

31. I have confirmed this conclusion by deriving something absurd (namely, perpetual motion!) from the opposite opinion—a commonly accepted one, especially among the Cartesians—that a body's force is proportional to the product of its size and speed. Using the same approach ·that I indicated in **25** and **26** above·, I have given an *a posteriori* definition of inequality of power. And at the same time I have shown how to distinguish clearly between a larger power and a smaller one, ·namely, as follows·. If the substitution of one force for another gives rise to perpetual mechanical motion, or an effect that is greater than its cause, then the two forces are clearly unequal; and the one that was substituted for the other must be the more powerful, since it produced something greater. I take it to be certain that nature never substitutes unequal forces for each other, and that the complete effect is always equal ·in power· to the total cause. So it is safe for us, •in our calculations, to substitute equal forces one for another with complete freedom, just as if we were actually substituting them •in reality; there is no risk that perpetual mechanical motion will result. ·With that in hand, I turn to my argument against the Cartesian view that force is proportional to (size × speed)·.

32. Most people have persuaded themselves that a heavy body A of a certain size and moving at a certain speed is equal in power to a heavy body C that is •half the size and moving •twice as fast. If they were right, then we could safely substitute either for the other ·in any physical set-up·. But this is not so. For suppose that two-pound body A has acquired one unit of speed by falling one foot. At that point, let us substitute for it a one-pound body C and have that moving upwards at an initial speed of two units. (On the Cartesian view this substitution ought to be all right, because on that view the two are of equal power: $2\times1 = 1\times2$.) But C, being launched upward at that initial speed, will rise *four* feet! Thus, simply by the one-foot fall of a two-pound weight, and the substitution of something supposedly of equal power, we have raised a one-pound weight four feet; and this is twice what we started with because 1×4 = **twice** 2×1. In this way we would achieve perpetual mechanical motion—which is absurd.

33. It doesn't matter whether we can actually *make* this substitution of C for A through the laws of motion; it is enough that we can substitute C for A in our thought-experiment because it is always valid to substitute things that are of equal power ·as the Cartesians say that A and C are·. Still, I have worked out various ways in which we *could* actually the transfer of A's whole force to C (or as near as you like to the whole of it), so that A is brought to rest and C starts moving. So it could actually happen that if they were equal in power, a two-pound weight with a certain speed could be replaced by a one-pound weight with twice that speed; from which, as I have shown, an absurdity would result.

34. These are not empty thoughts or mere verbal quibbles; they are very useful in comparing machines and comparing motions. Suppose you had enough force—from water, animals or whatever—to keep a 100-pound body in constant motion, so that it completed a horizontal circle thirty feet in diameter in 15 seconds; and suppose someone said he had a less expensive device that would make something twice the weight go around the circle in 30 seconds, ·implying that this would be an equally powerful upshot for a lesser expense·, you should realise that this would not be money-saving, and that you would be being tricked out of half your upshot-force. But now that I have disposed of the mistakes, let me set out a little more clearly the true laws—the *wonderful* laws—of nature.

Part 2: The laws of nature

35. The nature of body, and indeed of substance in general, is not well enough understood ·by the learned world·. As I have already mentioned, this has resulted in some distinguished philosophers of our time equating the notion of •body with that of mere •extension; which has driven them to bring in *God* in order to explain the union between soul and body, and even to explain how bodies can interact with other bodies. Now, it has to be accepted that pure extension—which has nothing to it except geometrical properties—could never be capable of acting and being acted on. So it seemed ·to these philosophers· that when a person thinks and tries to move his arm, God, as though by a prior agreement, moves it for him; and conversely when there is a motion in the blood and animal spirits God produces a perception in the soul. ·Given their premises, this was probably the conclusion they *had to* come to·. But because their conclusion is so far from good philosophical thinking, they should have realized that they were starting from a false principle—that a notion of body from which *these* consequences followed must be wrong.

36. I will show, therefore, that every substance has a •force for acting, and that every created substance also has a •force for being acted on. I will show too that the notion of extension in itself—·that is, considered purely geometrically·—is *not complete*. Extension is a relation to something that is extended; built into the ·complete· notion of extension is the notion of *something spread out or continuously repeated*. So extension presupposes this spread-out 'something', namely bodily substance, which has the power to act and resist, and which exists everywhere as bodily mass. Some day I shall use this to throw new light on the union of the soul and the body. But my present task is to show how it—·that is, this account of extension and body·—implies wonderful and extremely useful practical theorems in dynamics, which is the science dealing specifically with the rules governing forces in bodies. [In calling them 'rules' rather than 'laws', Leibniz may be following Descartes, who expressed his detailed physics of collisions in what he called 'rules'.]

37. The first thing to be properly grasped is that in substances, including created ones, **force is absolutely real** in a way in which •space, •time and •motion are not. Those three are in a way beings of •reason—·they are not things in the world but upshots of certain ways of •*thinking about* things in the world·. The only truth and reality there is to them comes from their involving •the divine attributes of immensity, eternity and activity, and •the force of created substances. It follows immediately from this that there is no empty place or time, and also that if we set aside *force* and consider *motion* purely in terms of the geometric notions of size and shape, and changes in them, motion is really nothing more than change of place. So **motion as we experience it is nothing but a relation**. Descartes recognised this when he defined *motion* as the removal of something from the neighbourhood of one body to the neighbourhood of another.

38. But in working out the consequences of this he forgot his definition, setting set up his rules of motion as if it were something real and absolute. Here is what has to be accepted:

> If a number of bodies are in motion, there is no empirical way of determining which of them are in absolute determinate motion and which are at rest. Choose any one you like as being at rest, and the ·empirical· phenomena will be the same.

Something follows from this that Descartes overlooked, namely that **the equivalence of hypotheses still holds when bodies collide**. ·Let me spell that out a little. Suppose that two bodies that are moving relative to each other collide; just before the collision we can say that

> (P_1) body A was stationary and body B was moving thus and so, or that
> (P_2) body B was stationary and body A was moving thus and so, or that
> (P_3) body A was moving thus and thus, and body B was moving so and so.

If these are properly formulated so that they do fit the phenomena—that is, do yield the observed *relative* motions of A and B—they are *equivalent hypotheses*. And my point is that if you have sound rules about what happens in collisions, the result of applying them to (P_1) should be exactly the same as that of applying them to (P_2) or to (P_3)·. So we must work out rules of motion that preserve the relative nature of motion; that is, there will be no way of determining from the phenomena after a collision which bodies before it had been at rest and which had been in absolute determinate motion. ·More strictly speaking: there *isn't* a way doing that; I am saying that one shouldn't have collision-rules implying that there *is*·.

39. So Descartes's rule according to which a body *at rest* can never be dislodged by a smaller body is a misfit, as are his other truth-deserting rules of the same kind. ·His trouble isn't confined to the concept of being at rest·. It also follows from the relative nature of motion that **the action or impact of bodies on each other will be the same, provided that the speed with which they come together is the same**. ·The crucial point there is that 'the speed with which they come together' involves relative, not absolute motion·. . . . And this is exactly what we find: we would feel the same pain if our hand knocked against a stationary stone hanging from a thread as when a stone hits our stationary hand with the same speed.

40. In practice we speak as the situation requires, giving—·as our choice from among the equivalent hypotheses·—the simplest and most suitable explanation of the phenomena.This is why we can appeal to the notion of a *first mover* in spherical astronomy, while in planetary theory we should use the Copernican hypothesis. (So the arguments that have been pursued so vigorously, drawing in even theologians, completely disappear. ·I mean arguments about such topics as whether the earth moves·.) For while force is something real and absolute, motion belongs to the class of relative phenomena; and the truth is seen not in the phenomena but in their causes.

41. Something further that follows from my notions of body and forces is that **whatever happens in a substance can be understood as happening spontaneously—·not caused from outside the substance·—and in an orderly way**. Connected to this is the proposition that **no change takes place in a jump**.

42. Given this, it also follows that **there cannot be atoms**. To see how that follows, think about two bodies that collide and rebound away from each other. If these bodies were atoms—that is, bodies of maximal hardness and inflexibility—then clearly their change of motion would be taking place in a 'jump', i.e. instantaneously, for the forward motion would have to change to backward at the very moment of collision. 'Perhaps the atoms might become stationary for an instant immediately after the collision, ·and *then* start moving in a different direction·.' That means that for a moment they lose all their force ·and then regain it·! Anyway, as well as containing this and other absurdities, this proposal would still involve a change taking place in a single jump—an instantaneous change from motion to rest, with no intermediate stages.

43. So we have to recognise that when two bodies collide, from the point of collision onwards they are gradually compressed, like two balloons, and as their motion towards each other continues the pressure increases *continuously*; that makes the motion decrease as the force of striving is converted ·from a force for motion· into ·a force for· the elasticity of the bodies, until they come to a complete standstill. Then, their elasticity begins to restore them, and they rebound from each other in the opposite direction; their motion begins from rest and *continuously* increases until they finally reach the same speed they had when they came together but in the opposite direction. . . .

44. As this account shows, none of these changes takes place in a single jump. Rather, the forward motion gradually lessens until the bodies are at rest, after which the backward motion begins. In just the same way, •one shape can't be turned into another—e.g. a circle into an oval—except by passing through all the countless intermediate shapes, and •nothing gets from one place to another, or from one time to another, without going through all the places and times in between. So •motion can never give rise to rest (let alone to motion in the opposite direction) without passing through all the speeds in between. Given how important this is in nature, I am amazed that it has been so little noticed.

45. It also follows that **•all rebounding arises from elasticity**. (Descartes rejected this in his letters, and some great men are still unwilling to allow it.) It explains many excellent experiments which show, as Mariotte has beautifully demonstrated, that **•bodies change shape before they bounce off anything**. And finally there is the most wonderful conclusion that •each body, however small, is elastic, and is permeated by a fluid consisting of bodies that are even smaller than it is. This means that **•there are no elements of bodies**, no perfectly fluid matter, and no unintelligible solid spheres of some supposed ‘second element’, of fixed and unchanging shape; on the contrary, the analysis ·of bodies· continues to infinity. [Here ‘no elements of bodies’ means ‘no smallest parts of bodies’. Perfectly fluid matter would be matter consisting of continuous *stuff* rather than an aggregate of smaller bodies. The phrase ‘second element’ is a reference to the physics of Descartes.]

46. In conformity with the **law of continuity**, which rules out jumps, •rest can be considered as a special case of •motion—that is, as vanishingly small or minimal motion—and •equality can be considered as a case of vanishingly small •inequality. So in formulating the laws of motion we shouldn’t need special rules for bodies that are •equal or for bodies that are •at rest; all we need are rules for bodies that are unequal (·including the special case of inequality = 0·) and rules for moving bodies (·including ones moving at speed = 0·). If we insist on having special rules for rest and equality, we must be careful that they square with the idea that rest is the limit of motion and equality is the smallest inequality; otherwise we will violate the harmony of things, and our rules won’t square with one another.

47. I first published this new technique for testing rules—mine and others’—in the *News of the Learned World* for July 1687. I called it a general principle of order, arising from the notions of infinity and continuity, and pointing to the axiom that the organization of an output is the same as the organization of the input. [In Leibniz’s formulation, these could be inputs and outputs not of a physical event but of a mathematical problem: the solution (‘that which is sought’) must preserve structural features of the data (‘the given’) that set the problem.] I stated it in general terms thus:

> **If in a series of cases the inputs approach each other continuously and eventually become the same, the consequences or outcomes must do so also.**

That’s how it is in geometry, where the •ellipse continuously approaches the •parabola: take one focus as fixed and move the other focus further and further away from it, then when it reaches infinity the ellipse becomes a parabola. Thus all the rules of the ellipse have to hold for the parabola, understood as an ellipse whose second focus is infinitely distant. So we can consider parallel rays striking a parabola as either coming from or going to the other focus. ·Moving back now from geometry to physics·: Consider a series of cases in which body A collides with the moving body B, the motion of A being the same in all the collisions; the motion of B is made smaller and smaller until eventually it becomes rest, and then turns into increasing motion in the opposite direction. I hold that when A and B are both in motion, the outcome (for each) of the collisions between them continuously approaches—and eventually becomes the same as—the outcome of the case where B is rest. So the case of rest, both in the inputs and in the outputs, is the limit of cases of motion along a line. . . .and so it is a special case of such motion.

48. When I tested the Cartesian rules of motion by this touchstone that I had brought across from geometry to physics, it brought to light an amazing jump that is quite contrary to the nature of things. [The next two sentences simplify Leibniz’s.] Take a series of collisions in which the inputs or ‘givens’ are varied *continuously*, and construct a diagram that describes what the outcomes are, according to Descartes’s rules. You’ll find that the line representing the motion of one of the colliding bodies ·through the series· is *not* continuous, but has amazing gaps and jumps about in an absurd and incomprehensible way. I also noted in that publication that the rules proposed by Father Malebranche didn’t entirely pass this test either. That distinguished gentleman reconsidered the matter, and has candidly acknowledged that this ·work of mine· had given him occasion to change his rules, which he published in a small book. I have to say, though, that he hasn't yet fully mastered this new technique, for there are still some things in his theory that don't quite fit.

49. [In what follows, Leibniz writes about a body's *passio*. That can mean 'passion', but in this context 'passive state or event' seems about right.] Something else wonderfully follows from what I have said, namely **that every passive state or event of a body is spontaneous—i.e. arises from an internal force—even if it is occasioned by something external**. I'm talking about the passive state that •belongs to the body itself, •results from the collision, and •is just the same whatever account we adopt of how motion and rest were distributed between the two bodies before the collision. For since the impact is the same, whatever the truth is about which body was moving (·supposing there *is* such a thing as the *truth* about this·), so *the effect* of the impact is equally distributed between them, and therefore **in a collision both bodies are equally active**—half the effect results from the action of one, and half from the action of the other. And since half the resulting effect (the passive state or event) is in one body, and half in the other, the passive state or event in either of them *can* be derived from its own action, with no *need* for anything to flow across into it from the other, even though the action of the one provides the occasion for the other to produce a change within itself. [On 'flowing across', see the explanation in **21** above.] ·There would be a need to explain B's reaction partly in terms of input from A only if it were a fact that A had had a bigger active role in the collision than B because it was moving faster than B; but we have seen that there is no such fact·.

50. When two bodies collide, their resistance together with their elasticity causes them to be compressed by the collision, and the compression in each is the same, whatever account we give ·of how motion and rest was distributed between them before the collision·. This is what we find in experience. Imagine two inflated balls colliding, with

both in motion or
one at rest ·and sitting on the ground·, or
one at rest and dangling from a string so that it can easily swing.

If their relative speed of approach is the same in each case, then the amount of compression or elastic tension in each will also be the same. Furthermore, when two colliding balls regain their shapes through the force of the elasticity or compression they contain, each one drives the other away, so that they shoot out like arrows from a bow, each driving itself away from the other by force, receding from the other through its own force and not through the equal force of the other. What is true of these inflated balls must be understood as applying to all passive states and events in a colliding body: its rebound arises from its own elasticity—that is, from the motion of the ethereal fluid matter by which it is permeated—and so from a force existing inside it. (As I have said, I have in mind here the ·relative· motions of the bodies taken individually, as against the motion they all share, which we can describe as the motion of the centre of gravity of the lot of them taken together. ·To think of both kinds of motion at once·, think of these bodies as all being carried on a ship, whose motion is that of their common centre of gravity, while they move around *in* the ship, changing their relations to one another. The facts can all be covered by an account in which the common motion of the ship is ·suitably· put together with the individual motions of the bodies.) From what I have said we can also see that **there is no action of**

bodies without a reaction, and that they are equal and in opposite directions.

51. Since at any moment only force and its resultant effort exist (for, as I explained ·in **37**· above, motion never really exists), and since every effort tends in a straight line, it follows that **all motion is in a straight line or is composed of motions in straight lines**. Not only does it follow from this that **anything moving in a curve strives always to go off ·the curve and· along the straight tangent**, but also—utterly unexpectedly—there follows **the true notion of solidity**. In fact nothing is absolutely solid or absolutely fluid; everything has a certain degree of both solidity and fluidity, and whether we call a thing 'solid' or 'fluid' depends on the overall appearance it presents to our senses. My topic here is the sort of thing we call 'solid', ·not implying that its solidity is anything more than a matter of degree·. Consider such a thing rotating about its centre: its parts will be striving to fly off along the tangent; indeed they will actually begin to do so. But as each one's moving away from the others interferes with the motion of its neighbours, they are pushed back together again, as if a magnetic force at the centre was attracting them, or as if the parts themselves contained a centripetal force. Consequently, the rotation is composed of •the straight-line effort of the parts along the tangent and •their ·straight-line· centripetal striving among themselves. Thus we see that •all motion along a curve arises from the composition of straight-line efforts, and at the same time that •all solidity is caused by this pushing together by surrounding bodies—otherwise it couldn't be the case that all motion in a curve is made up of straight-line motions. This also gives us another—equally unexpected—argument against atoms.

52. [This section supplies—in a manner that can't conveniently be shown by ·dots·—some details that Leibniz omits from his extremely compressed exposition.] Nothing more contrary to nature can be imagined than to think—as Descartes did—that solidity derives from rest, for **there is never any true rest in bodies**; but I shall show that even if there *were* true rest, the most it could conceivably explain is a wholly resting body's •resistance to being moved (nothing but rest could come from rest!), and it wouldn't explain •solidity. Suppose that A and B are parts of a wholly resting body, so that they are at rest with respect to each other (which of course they couldn't be, because **no body ·or part of a body· ever keeps exactly the same distance from another for even the shortest time**.) Let it be granted that a thing at rest will remain at rest unless some new cause comes along to start it moving. And let Descartes have this as the explanation of why B •resists being moved when another body strikes it. Even if the facts about rest explained that much, they still wouldn't explain •solidity, because they don't explain why

> when B is struck, and moves, *it drags A along with it*;

and *that* is what would be needed for an explanation of the solidity of the body of which A and B are parts. A's following along with B would be explained by forces of attraction if there were such things, but there aren't. So, failing attractive forces, and failing the attempt to base solidity on rest, we are left only with the idea—which I endorse—that a body's solidity should be explained only in terms of its being pushed together by surrounding bodies. It can't be explained just by *grabbing*, with B being held back by A alone. [Following Garber's suggestion that Leibniz's *pressio* ('pressure') was a slip for *prensio* ('arrest' or 'seizure'). As a point about seizing or grabbing, what Leibniz says is intelligible; as a point about pressure, it isn't.] What we have to understand is that A and B *do* in fact separate from each other but are then driven back towards each other by the surrounding bodies. So their togetherness as parts of a single body results from the combination of these two motions.

53. Some people ·try to· explain the solidity of hard, perceptible bodies by supposing there to be imperceptible slabs or layers in bodies—like two slabs of polished marble that fit perfectly together—which the resistance of surrounding bodies makes it difficult to separate. Although much of what they say is true, it can't provide the basic explanation of solidity as such, because it assumes that the plates are themselves solid ·and does nothing to explain what *their* solidity consists in·.

54. All of this shows why I can't agree with some of the philosophical opinions of certain important mathematicians—·especially Newton·—who not only allow empty space and seem not to object to ·forces of· attraction, but also maintain that motion is something absolute, and claim to prove this through rotation and the centrifugal force arising from it. ·Their 'proof' doesn't work, however·. When we are dealing with straight-line motions, the equivalence of hypotheses is preserved; that is, we get the same result whichever object we take to be the moving one—·see **38**· above. Well, the same thing holds for movements along a curve—e.g. rotations—because every curved motion arises from a combination of straight-line motions.

55. From what I have said it can be understood that **motion that is common to a number of bodies doesn't change their actions on one another**, because the speed with which they collide and therefore also the force of impact in their collisions is not altered ·by whatever movement the whole system of them is undergoing·. This is what lies behind some fine experiments on motion that Gassendi reported in his letters; they concerned motion imparted by something which is itself being carried [Leibniz seems to have meant: 'motion imparted by one thing to another when they are *both* being carried'], as a reply to those who thought they could infer from the motion of projectiles that the earth doesn't move. Consider people travelling in a big ship, in conditions in which they can't see anything outside it. Clearly, as long as the ship moves smoothly and uniformly—whatever its speed—the events *within* the ship won't tell them whether or not it is moving, even if they move about and throw balls.

56. This should be noted because ·of its relevance to a certain mistake people have made about why, according to the Copernican theory, things launched straight upward fall back to where they started from, although during that time the earth has rotated a little·. Some people have got the Copernican theory wrong: they take it to be saying that when a thing is thrown straight upward, it is carried along •by the air that is rotating with the earth, and *that* is why it falls back to where it was, as though the earth were not moving. They rightly judge that this is not acceptable. However, the ablest users of the Copernican theory ·don't appeal to the effect of the rotating air; rather, they· believe that because whatever is on the surface of the earth is moving along with it, when a thing is launched from a bow or catapult it carries with it •the impetus it has received from the motion of the earth as well as •the impetus it got by being launched. So the projectile has two motions, one in common with the earth and one special to the launching, and it's not surprising that the common motion doesn't make any difference ·to how the projectile relates to the place from which it was launched·. ·This is correct only within limits·. If a projectile were thrown up far enough,. . . .the place P from which it was launched would have moved far enough *around its arc* for this ·little journey· to be noticeably different from one *in a straight line*; and in that case the projectile wouldn't fall back precisely onto P, because its only motion other than the up-and-down one is the *straight-line* motion given to it by the rotation of the earth.

57. The effort of heavy bodies to move towards the centre ·of the earth· brings in an external influence that can also make a difference in the phenomena; another relevant factor would be provided by a compass in the enclosed ship which by pointing always to the pole could indicate when the ship changed course. But ·these facts are not objections to what I have been saying. They merely serve to warn us that· when we are concerned with the equivalence of hypotheses, *everything* that plays a part in the phenomena must be taken into account. All this shows that we can safely combine several motions into one, and resolve one motion into several. . . . But the matter certainly needs to be proved, and can't simply be taken as self-evident, as many have done.